Spread Too Thin

Managing Coefficient of Variation in Monte-Carlo Based Cost Models

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Speaker Bio

- **Stephen Koellner** – Analyst/Technical Advisor
  - 5+ years of experience in federal project cost estimation
  - PLCCEs/POEs, IGCEs, Cost-Benefit Analysis, AoAs
  - ICEAA 2022 Team Achievement of the Year Award recipient
  - GAO Cost Guidebook contributor
  - Published author: Earth & Planetary Science Letters
  - BS in Mathematics - Penn State University

- Augur Consulting Inc.
  - Service-Disabled Veteran Owned Small Business (SDVOSB)
  - Supporting our government-only (DOD & DOE) customer base since 2012
  - Core Competencies:
    - Cost Estimating and Analysis
    - Integrated Master Scheduling
    - Performance Management
    - Data Analytics and Visualization

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Agenda

- Introduction
- Cost Uncertainty Modeling Methods
- Coefficient of Variation
- Interpretation of a WBS as a Convolution
- Behavior of Top-Level CV
- Randomly Generated WBS
- Recommendations
- Conclusion

Introduction

- **Problem:** Uncertainty easily underestimated in cost models
  - Inaccurate quantification of cost spread poses significant long-term risk
  - Characterized by a low Coefficient of Variation (CV)
  - Diagnosing issue often a difficult endeavor

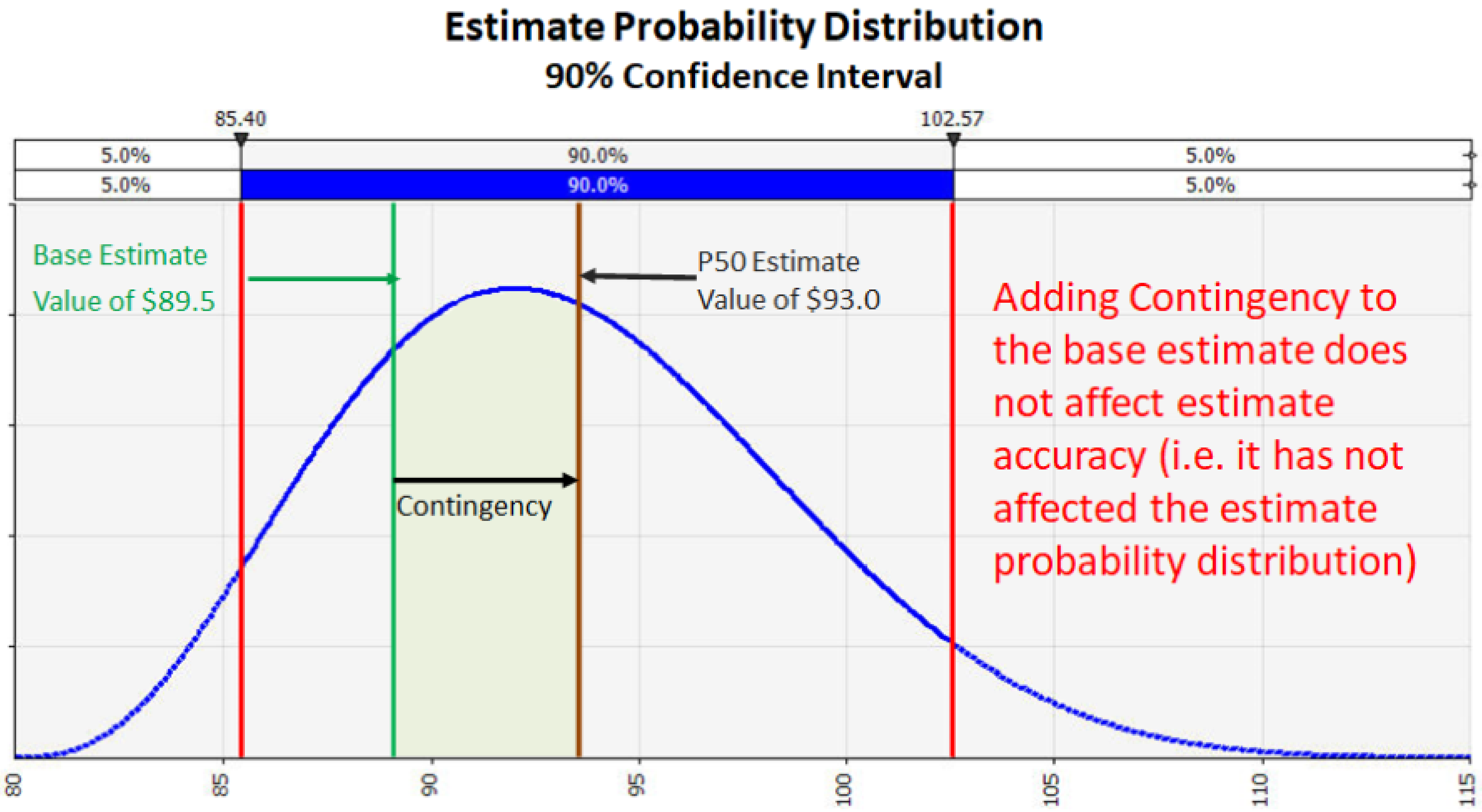
- **Goal:** Identify modeling choices that prohibit realistic cost spread
  - Define CV as function of children elements in WBS
  - Study interactions of input level uncertainty & output level uncertainty
  - Provide modeling guidelines to cost estimators
  - Enable program managers to minimize likelihood of funding risks

- **Ground Rule:** Topic only covers cost uncertainty
  - Schedule uncertainty & risk events are future endeavors of this analysis

Cost Uncertainty

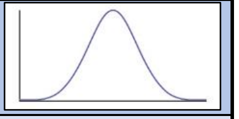
- AACE International RP 104R-19:
  - *"… estimates are predictions of an uncertain future, it is a recommended that all estimate results should be presented as a probabilistic distribution of possible outcomes in consideration of risk."*
- All cost estimates should account for risk/uncertainty
  - Credible cost models produce a range (spread) of values
  - Cost modelers must primarily think of output as a distribution, not a number
  - Often brief a snapshot of distribution/spread to clients
- Analyze Results
  - Determine if cost output logically aligns with cost inputs
  - Evaluate if top-level cost uncertainty adequately matches program status
  - Identify cost drivers and quantify their impact

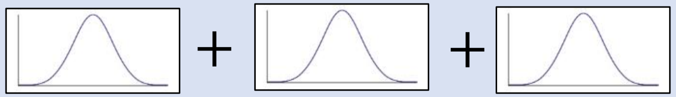
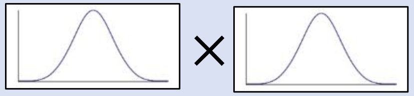
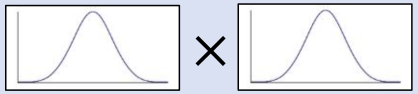
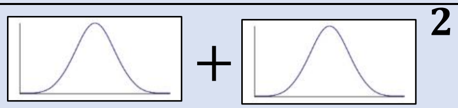
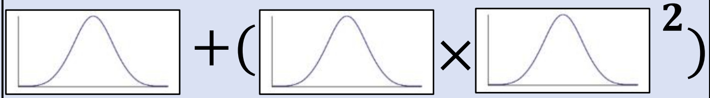
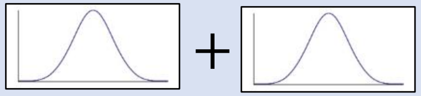
Cost Uncertainty



*From RP 104R-19

Application of Cost Uncertainty

Top - Down Application of Cost Uncertainty			
WBS Level	**WBS Element**	**Equation**	**Application of Risk**
1	**Total Contract Cost**	*Sum of Children*	
2	Management	*Labor Pool 1 + Labor Pool 2 + Labor Pool 3*	*None*
2	Development Labor	*Headcount x Labor Rate*	*None*
2	Prototype Materials	*Quantity * Unit Cost*	*None*
2	Equipment	*Base Cost + Complexity Factor* ^{2}	*None*
2	Testing Labor	*Historical Cost + (Factor x Test Quantity* ^{2} *)*	*None*
2	Testing Equipment	*Equipment 1 + Equipment 2*	*None*

Bottom - Up Application of Cost Uncertainty			
WBS Level	**WBS Element**	**Equation**	**Application of Risk**
1	**Total Contract Cost**	*Sum of Children*	*Composition of Children*
2	Management	*Labor Pool 1 + Labor Pool 2 + Labor Pool 3*	
2	Development Labor	*Headcount x Labor Rate*	
2	Prototype Materials	*Quantity * Unit Cost*	
2	Equipment	*Base Cost + Complexity Factor* ^{2}	
2	Testing Labor	*Historical Cost + (Factor x Test Quantity* ^{2} *)*	
2	Testing Equipment	*Equipment 1 + Equipment 2*	

Application of Cost Uncertainty

Comparing Application Methods of Cost Uncertainty		
Application	**Pros**	**Cons**
Top - Down	• Simplifies cost modeling • Generally, more data is available to defend top level spread	• Limited ability to analyze cost drivers and quantify impact to model spread • Assumptions on spread not directly traceable to inputs Range of cost outcomes can only be viewed at top-level
Bottom - Up	• Spread of total cost directly depends on cost inputs • Range of cost outcomes can be viewed for any WBS element	• Complicates cost modeling/behavior of cost model • Can more easily underestimate cost uncertainty

- Both types of application have unique strengths and weaknesses
  - Choice depends on agency guidance, estimate type, estimator preference
  - E.g., ROM estimates may employ use of Top-Level application
  - Augur typically develops estimates with Bottom-Level application
- Bottom-level requires approximation methods (Monte Carlo)
- Brief evaluates behavior of Bottom-Level application of spread

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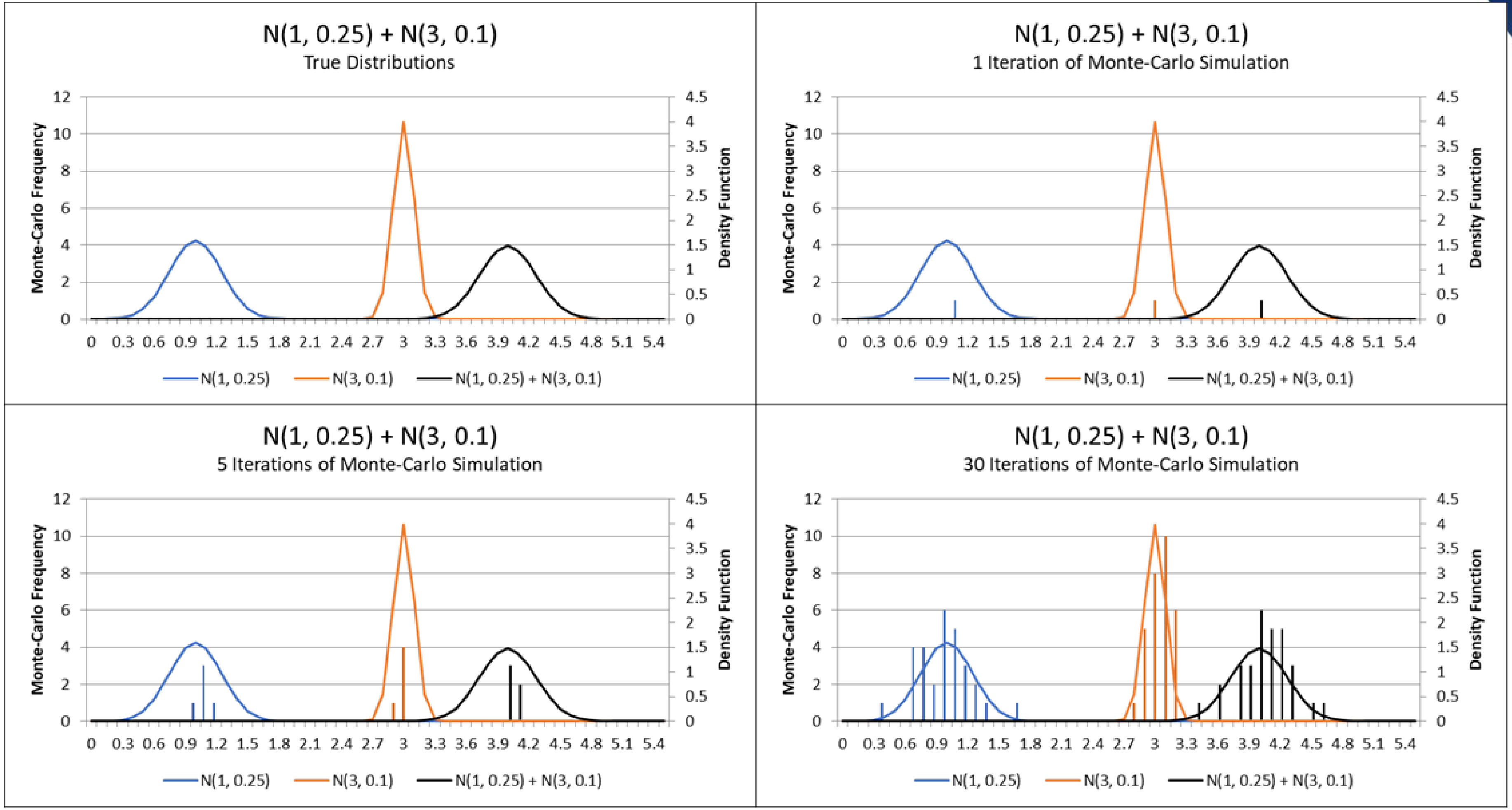
Monte Carlo Cost Modeling

- Calculate cost outputs with Monte Carlo sampling
  - Interactions of distributions are incredibly complex
  - “By hand” calculations impractical & inefficient
- Monte Carlo models approximate outputs efficiently
  - Sample random values from input distributions
  - Run calculation of outputs/save results from this iteration
  - Results converge to true value as over iterations

Monte Carlo Cost Modeling



Coefficient of Variation

$$CV_X := \frac{\sigma_X}{\mu_X}$$

- Why is CV important to cost estimators?
  - CV is a ratio that "normalizes" the spread of a distribution
  - Allows comparison of data sets with differing means/standard deviations
  - Commonly used to check if uncertainty is appropriately captured in model
  - Higher CV indicates a wider dispersion/flatter distribution

Interpretation of WBS as a Convolution

- Work Breakdown Structure (WBS)
  - Higher level elements (parent) are sum of lower level/subordinate elements (children)
  - All WBS elements are just probability distributions
  - Convolution = Linear combination of probability distributions
- Let Z be a parent-level WBS element with n children elements: X_i

WBS Level	WBS Element
1	**Total Contract Cost**
2	Management
2	Development Labor
2	Prototype Materials
2	Equipment
2	Testing Labor
2	Testing Equipment

→

WBS Level	WBS Element
k	**Z**
k+1	X_1
k+1	X_2
k+1	.
k+1	.
k+1	.
k+1	X_n

$$Z = \sum_{i=1}^{n} X_i$$

- CV of Z can be defined in terms of its children
  - Approx. computationally in using Monte-Carlo simulation SW

Top Level CV Equation

- Each X_i is a distribution with parameters:
  - $r_{i,j}$ are correlation coefficients between X_i and X_j
  - μ_{X_i} is the expected value (mean) of distribution X_i
- The CV of parent level Z follows the below equation:

$$Z = \sum_{i=1}^{n} X_i$$

$$CV_Z = \frac{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} r_{i,j}\, CV_{X_i}\, \mu_{X_i}\, CV_{X_j}\, \mu_{X_j}}}{\sum_{i=1}^{n} \mu_{X_i}}$$

- Equations are agnostic to types of distributions

Verification of CV Equation

- Monte Carlo simulation approx. uncertainty distributions
  - Correlation is simplified via group strength
  - Small deviations between model & equation (depends on # of iterations)
- Formula is not useful for generating cost output
  - Cost models are more complex than simple sums
  - Equation is useful for analyzing results at WBS level

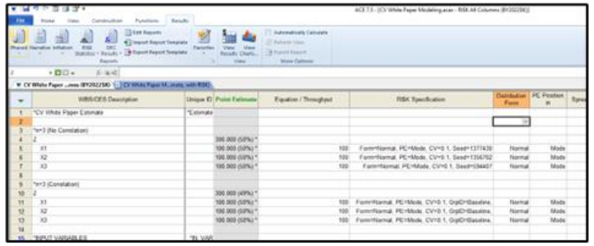
Simplified WBS (n=3)			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

CV Calculation	
True CV (Eqn)	0.0707
Monte Carlo Sim*	0.0710
% Difference	-0.41%

** 10,000 Iterations*

$$CV_Z = \frac{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} r_{i,j}\, CV_{X_i}\, \mu_{X_i}\, CV_{X_j}\, \mu_{X_j}}}{\sum_{i=1}^{n} \mu_{X_i}}$$



Behavior of Equation

- Illustrate behavior of top-level CV
  - Change one parameter of baseline for each scenario
  - Maintain perturbations proportionally
  - Identify how children elements impact parent level

- Modeling choices vs baseline updates

Simplified WBS (n=3)			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

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Perturbation – Increased CV of WBS Element

- Larger spread of lower elements increases parent spread
  - Double standard deviation of one child element
  - Parent CV increase from 0.071 to 0.097
  - ~37% increase in parent CV
- Intuitive result, large impact for small WBS
  - Average CV of children elements substantially higher

Baseline WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Increased CV of WBS Element			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	20	0.2
X2	100	10	0.1
X3	100	10	0.1

CV Calculation	
Baseline	0.070711
Increased CV of WBS Element	0.097183
%Δ CV	37%

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Perturbation – Large Mean (Normalized)

- Double mean of one element/scale others proportionally
  - Represents a ~12% increase to top level CV
  - Same increase as the normalized standard dev. case
    - Again, "grouping" spread to single element
    - Same top-level distribution, despite different children

Baseline WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Large Mean (Normalized)			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	200	20	0.1
X2	50	5	0.1
X3	50	5	0.1

CV Calculation	
Baseline	0.070711
Large Mean (Normalized)	0.079057
% Δ CV	12%

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Perturbation – Large WBS

- Double size of WBS, maintain same total sum
  - Large WBS case reduced top level CV by ~13%
  - Increased model fidelity dramatically reduces spread of costs
  - More detailed and precise estimate =/= more accurate estimate
  - Example: quantity takeoff used in early planning estimate

Baseline WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Large WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	50	5	0.1
X2	50	5	0.1
X3	50	5	0.1
X4	50	5	0.1
X5	50	5	0.1
X6	50	5	0.1

CV Calculation	
Baseline	0.070711
Large WBS	0.061237
% Δ CV	-13%

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix				
	X1	X2	...	X6
X1	1	0.25	0.25	0.25
X2	0.25	1	0.25	0.25
...	0.25	0.25	1	0.25
X6	0.25	0.25	0.25	1

Perturbation – Stronger Correlation

- Double a single correlation coefficient
  - X_1 and X_3 are 2 times more correlated than other pairs
  - Top-level spread increased by ~5%
- Increasing correlation increases top-level CV

Baseline WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

Strong Correlation			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

CV Calculation	
Baseline	0.070711
Strong Corr	0.074536
% Δ CV	5%

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.5
X2	0.25	1	0.25
X3	0.5	0.25	1

Perturbation – No Correlation

- Absence of correlation
  - Dramatic reduction in top level spread: ~18%
  - Effectively independent distributions being summed
- Zero correlation is unrealistic
  - Bottom-level application of dist. transfers correlation to WBS
  - Inter-dependence of common inputs creates functional correlation
- Note: negative correlation will also reduce top level CV

Baseline WBS			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

No Correlation			
WBS	Mean	Stan. Dev.	CV
Z	300		
X1	100	10	0.1
X2	100	10	0.1
X3	100	10	0.1

CV Calculation	
Baseline	0.070711
No Corr	0.057735
% Δ CV	-18%

Correlation Matrix			
	X1	X2	X3
X1	1	0.25	0.25
X2	0.25	1	0.25
X3	0.25	0.25	1

Correlation Matrix			
	X1	X2	X3
X1	1	0	0
X2	0	1	0
X3	0	0	1

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Summary of Perturbations

- Cost variable/cost data characteristics: mean, standard deviation, & correlation
- Cost modeling choices
  - How much scope a single WBS element captures (grouping)
  - The level of detail of the cost model
  - Correlation applied in absence of data
- As correlation **increases ↗** the top level CV also **increases ↗**
- As WBS size **increases ↗** the top level CV **decreases ↘**

Behavior of CV			
Scenario	**CV**	**% Δ to Baseline**	**Note**
Baseline	**0.0707**	**0%**	***n=3, μ = 100, CV = 0.1, r = 0.25***
High Stan. Dev.	0.0972	37%	*Double one standard dev.*
Large Mean	0.0729	3%	*Double one mean*
Large Mean (Normalized)	0.0791	12%	*Double one mean, reduce mean of other elements*
Large WBS	0.0612	-13%	*Double WBS/maintain top-level mean*
Strong Correlation	0.0745	5%	*Double single correlation coefficient*
No Correlation	0.0577	-18%	*Model independent distributions*

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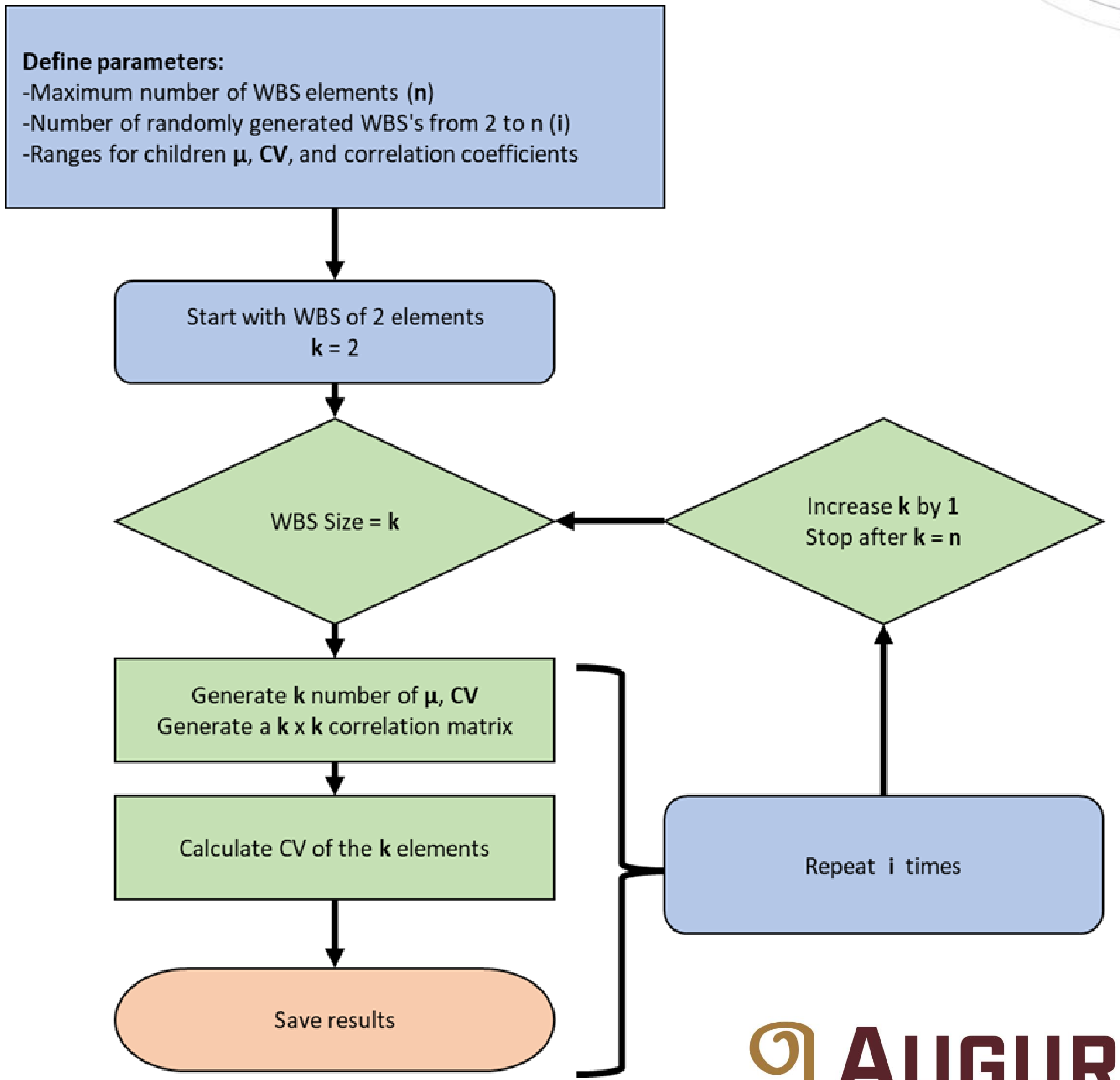
Randomized WBS

- Formula can be used to model cost estimator behavior
  - Randomly generate WBS's and calculate top-level spread
  - Follow best practices to provide recommendations for analysts
  - Refined simulations of WBS parameters
    - More precisely model mean, children CV's, correlation, etc.
    - E.g., Children CV from uniform distribution between 0.15 – 0.6
- Randomly generate WBS
  - Common values of:
    - μ, CV, & Correlation Coefficients*
- Study CV behavior at scale
  - Simulate practices of cost estimators
  - Model impacts of correlation at the WBS level

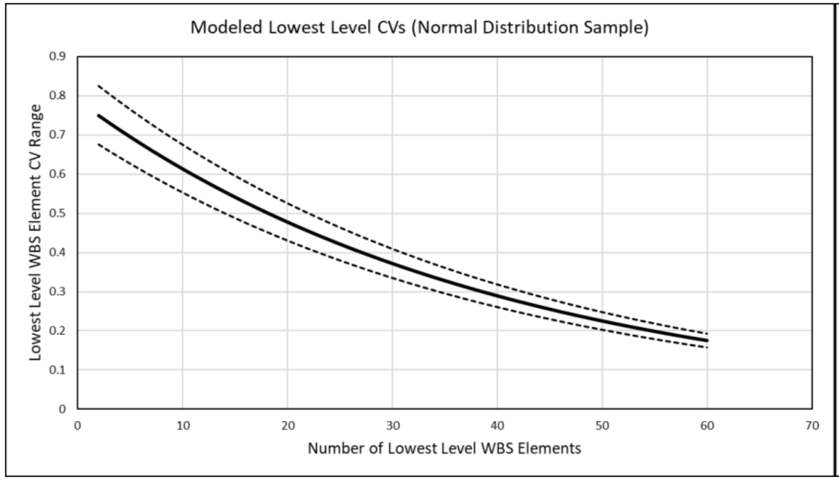
*Values scale logically as WBS size increases

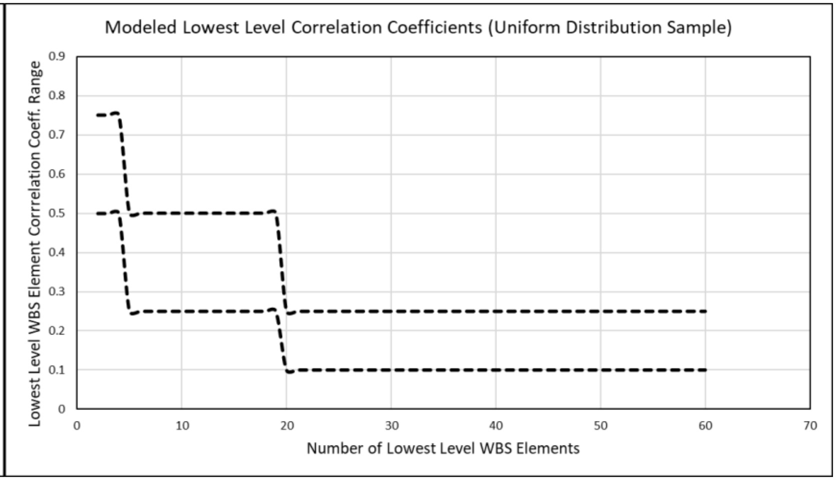
Randomized WBS



Randomized WBS

- Randomly generate statistics for children elements
  - Children CV sampled from a normal distribution
    - Mean and standard dev. decrease with more children elements
    - Higher fidelity -> less uncertainty on individual elements
  - Correlation coefficients sampled from uniform distribution
    - Step down in line with USAF guidance on correlation coefficients





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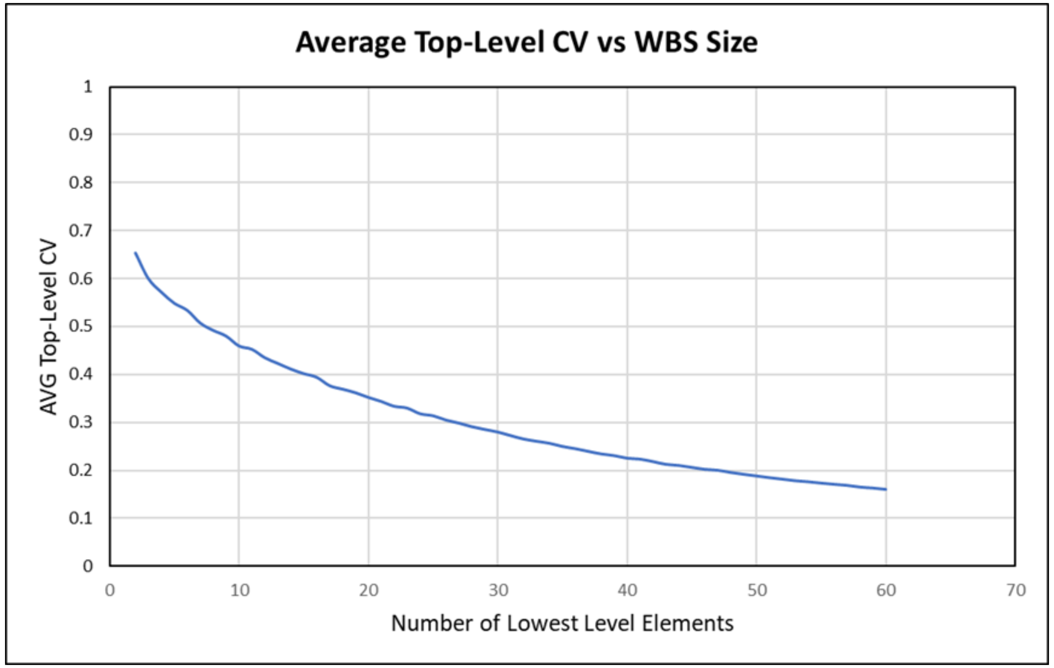
Randomized WBS

- Random WBS designed to mimic ACAT I cost model
  - WBS totals between \$3.6B - \$5.8B
    - Choice of WBS sum is irrelevant for exercise
  - Children normalized to have sum within above range



Commonly Accepted Ranges for CV

- Below are proposed ranges from USAF IT research paper
  - CV ranges by acq. milestone (based on actual cost growth)
  - Ranges are preferable since they are traceable to actual data

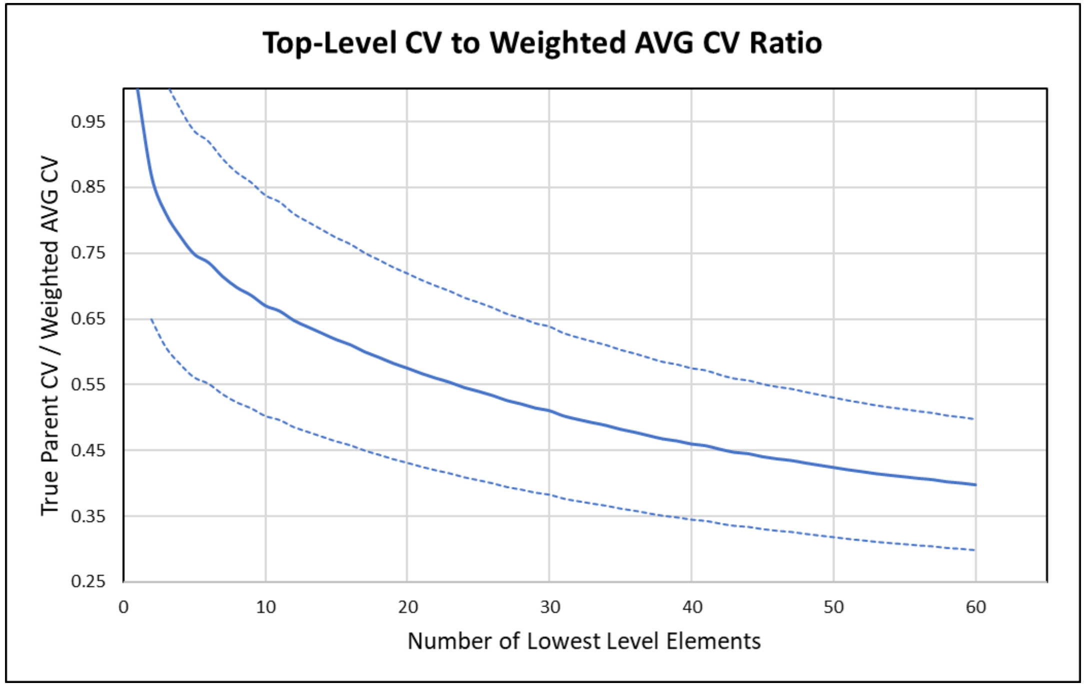
USAF IT Research Paper	
Estimate Type	**CV Range**
Milestone A	0.41 - 0.74
Milestone B	0.31 - 0.54
Milestone C	0.23 - 0.32

- Ranges & randomized WBS results used for WBS size rec.
  - Compare at-scale CV behavior with ranges to make rec.
  - CVs should not be the only statistic analyzed for model health

Recommended WBS Ranges	
Acquisition Phase	**Rec. WBS Size**
Milestone A/High Uncertainty	2 - 14 Lowest Level Elements
Milestone B/Medium Uncertainty	6 - 25 Lowest Level Elements
Milestone C/Modest Uncertainty	24+ Lowest Level Elements

Ratio of True CV to Weighted Avg CV

- Ratio curve based on following modeling best practices
  - Significant deviations from this curve indicate lack of correlation
    - E.g., a WBS with 20 elements w/ratio of 0.3
    - Ratio should be ~0.55 -> correlation coefficients too low



CV in Cost Estimating Applications

- Analyze output CV as a sanity check for cost model spread
  - Check ratio of top-level CV and weighted AVG CV w/WBS size
  - Check dollar value spread of outputs for reasonableness
  - CV should NOT be the only metric used for evaluation

- Observable CV behavior provides cost modeling insight
  - Early ROM estimates need small WBS/top-level risk application
  - Ensure appropriate correlation is being applied to input variables
  - WBS size should correlate with program maturity and level certainty
  - Don't over sharpen the pencil with engineering build-ups

- **Leadership should push for higher spread in early estimates**
  - Funding requests need accurate projections of potential cost growth
  - Underestimated spread reduces contingency in risk informed cost models

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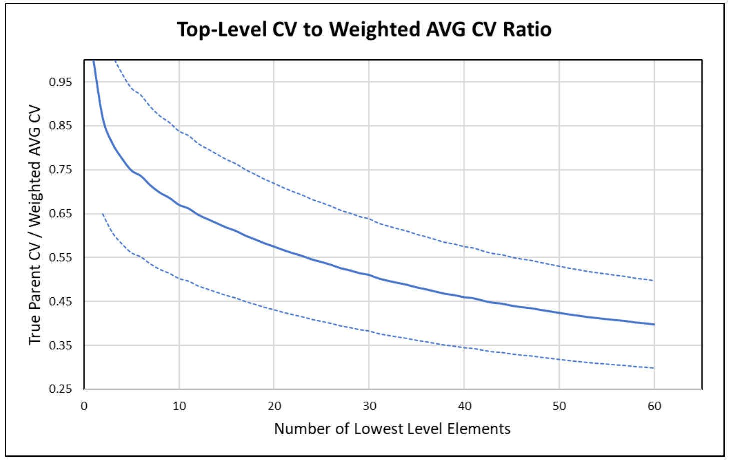
Conclusion

- Insufficient cost spread in Monte-Carlo based cost models
  - CV equation provides insight to understanding top-level CV behavior
  - Provided rules of thumb/cross-checks for diagnosing cost models
- **Top-level CV dominated by WBS size and correlation**
  - Models w/out correlation are underestimating spread
  - WBS size should fall within ranges based on lifecycle/certainty level
  - Overly detailed WBS injects overoptimism unless correlated properly

Recommended WBS Ranges	
Acquisition Phase	**Rec. WBS Size**
Milestone A/High Uncertainty	2 - 14 Lowest Level Elements
Milestone B/Medium Uncertainty	6 - 25 Lowest Level Elements
Milestone C/Modest Uncertainty	24+ Lowest Level Elements



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